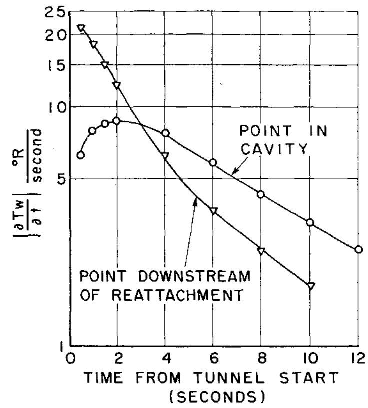

Fig. 1 Typical temperature-history data from cavity model

there with a very small value. Downstream of the cavity, however, the model surfaces changed temperature quickly (particularly in the reattachment region), and the result of this was to draw heat from the cavity surfaces. (Initial wall temperature was greater than adiabatic wall temperature in these experiments.) Later, when some measure of balance had been attained between convection and conduction terms, the entire model continued more slowly to the recovery temperature.

This explanation was checked by carrying out calculations of the conduction terms in the heat transfer equation for the model skin, using polynomials to fit the data at times after the tunnel start. Using these complete calculations, the heat transfer coefficients were calculated over a finite time interval in the early part of a run, and the values were compared with that given by the *initial* gradient of the temperature-time trace. Good agreement was obtained in all cases.

In view of this, it is suggested that caution be exercised in reducing data from experiments using the transient technique in cases where the heat transfer rate varies greatly along the model. Fitting a curve through the data in order to obtain initial temperature gradients is only valid if the form of the curve is known beforehand, and in extreme cases, an exponential variation is not a valid assumption. The need for caution is made greater because of the diffusive nature of the heat transfer equation governing the model temperature distribution. It was found in the present experiments that the temperature traces became roughly exponential in character a few seconds after the tunnel start. This is due to the properties of the parabolic diffusion equation and has no connection with the initial heat transfer rates. There is some danger that an investigator accustomed to exponential temperature traces from more conventional configurations might ignore the initial peculiarities of the curves as due to some initial unsteady effect and use the later exponential sections of the curves for extrapolation purposes. In the case of the present experiments, this would have resulted in the separated-zone heat transfer rates being overestimated by a factor of three. This comment applies particularly to the use of "point-record" potentiometers for recording thermocouple outputs. If the gap between measurement points is too large, the important part of the curve may be missed althogether.

It should be noticed that the initial starting process in a separated-flow configuration may take longer than for an attached flow, because initial transient conditions are not swept immediately downstream. The characteristic time for setting up the steady-state configuration in a cavity flow is of the order of D^2/ν for diffusion of vorticity and D^2/α for diffusion of heat, where D is the depth of the cavity, ν the kinematic viscosity, and α the thermal diffusivity of the fluid. These times were found to be of the order of milliseconds in the present experiments, and the starting process therefore was rejected as a cause of the initial peculiarities in the behavior of the temperature-time traces. (The point of inflexion in the trace from a point in the cavity was found to occur from 2 to 4 sec after the tunnel start.) A more complete discussion of the foregoing is contained in a paper by Nicoll.[1]

[1] Nicoll, K. M., "The use of the transient 'thin-wall' technique in measuring heat-transfer rates in hypersonic separated flows," Princeton Univ. Dept. of Aeronaut. Eng. Rept. 628 (July 1962).

Delaying Effect of Rotation on Laminar Separation

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THE experiments of Himmelskamp[1] on an airscrew suggest that the rotation postpones stalling to a higher lift coefficient than would be expected from the two-dimensional characteristics of the airscrew blade sections. In confirmation of the experiments, this note presents a theoretical analysis of a simple case where rotation is found to delay laminar separation and sometimes to prevent it entirely.

Consider a helical surface rotating and advancing at zero incidence, with a straight leading edge perpendicular to the axis of rotation, as in Fig. 1. It is shown in Ref. 2 that the equations for the boundary layer on such a surface are, if the pitch is only moderate, approximately equivalent to those for zero pitch, i.e., for a flat sector of a circle rotating in its own plane, with zero velocity along its axis of rotation. Referred to axes θ, r, y rotating with the sector with angular velocity Ω, the velocity components u and w in the tangential and radial directions are Ωr and zero outside the boundary layer. Suppose now that the helical surface, to which the sector is equivalent, is distorted slightly or operated at a small incidence, so that the pressure over its surface is nonuniform. Consider the special simple case where, for the equivalent sector, the tangential and radial velocity components outside the boundary layer are $\Omega r(1 - K\theta)$ and zero, where K is a constant. The equations of motion become approximately

$$\frac{u}{r}\frac{\partial u}{\partial \theta} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial r} - \frac{w(2r\Omega - u)}{r} = -r\Omega^2 K(1 - K\theta) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{u}{r}\frac{\partial w}{\partial \theta} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial r} - \frac{(r\Omega - u)^2}{r} = -r\Omega^2 K^2\theta^2 + \nu\frac{\partial^2 w}{\partial y^2} \quad (2)$$

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial r} + \frac{w}{r} = 0 \quad (3)$$

Received by IAS November 13, 1962. This work was carried out in the Ship Division of the National Physical Laboratory, and this note is published with the permission of the Director of the Laboratory. R. S. Martin of the Mathematics Division of the National Physical Laboratory was of great assistance in the numerical integration of the equations.

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Table 1 Effects of varying K

K	∞	6.8	1.4	1.0	0.9	0.8	0.7	0.6	0.55
ψ_s	0.136	0.136	0.139	0.143	0.145	0.148	0.153	0.165	0.185
θ_s	0.0	0.020	0.100	0.143	0.161	0.184	0.218	0.274	0.336
K	0.55	0.50	0.45	0.40					
$(\psi\eta_1^2)_{\max}$	3.02	2.63	2.29	1.98					

where

$$u = v = w = 0,\ y = 0$$

$$u \to r\Omega(1 - K\theta),\ w \to 0,\ y \to \infty \tag{4}$$

Put $y = (2\theta\nu/\Omega)^{1/2}\eta$, and $\gamma_1 = \eta/\eta_1$, $\gamma_2 = \eta/\eta_2$. Assume that approximately

$$u = r\Omega(1 - K\theta)\left[\frac{3\gamma_1}{2} - \frac{\gamma_1^3}{2} - \frac{K\theta\eta_1^2}{2}\gamma_1(1-\gamma_1)^2\right],\quad 0 \leqslant \gamma_1 \leqslant 1$$

$$= r\Omega(1 - K\theta),\ \gamma_1 \geqslant 1 \tag{5}$$

and

$$w = (r\Omega\theta/2)\eta_2^2(1 - K^2\theta^2)(\gamma_2 - 2\gamma_2^2 + \gamma_2^3),\ 0 \leqslant \gamma_2 \leqslant 1$$

$$= 0,\ \gamma_2 \geqslant 1 \tag{6}$$

These expressions satisfy Eqs. (1) and (2) at the surface $y = 0$, together with the boundary conditions in Eq. (4). If they are substituted into the equations obtained by integrating (1) and (2) with respect to y across the boundary layer [making use of Eq. (3)], the resulting equations are of the following form:

$$d\eta_1/d\psi = F(\psi, \eta_1) + (1/K^2)G(\psi, \eta_1, \eta_2) \tag{7}$$

and

$$d\eta_2/d\psi = H(\psi, \eta_1, \eta_2)(d\eta_1/d\psi) + I(\psi, \eta_1, \eta_2) + (1/K^2)L(\psi, \eta_1, \eta_2) \tag{8}$$

where $\psi = K\theta$. These equations have been integrated concurrently for various values of K by a step-by-step process, using the Ace computer of the Mathematics Division of the National Physical Laboratory. If the solution indicates that $\psi\eta_1^2$ reaches the value 3, then at this point Eq. (5) shows that $\partial u/\partial y$ is zero at the surface, implying separation of the u component profile. If K is very large and ψ is finite at separation, this means that θ at separation must be very small, and the rotation can have had no appreciable effect on the flow. Thus when $K \to \infty$, so that the terms in G and L of Eqs. (7) and (8) vanish, the former equation becomes equivalent to that for a two-dimensional flow with a linear adverse external-velocity gradient, as solved by Howarth.[3] The accurate numerical solution of this problem has $\psi = 0.120$ at separation. Using step lengths in ψ of 0.01 and 0.005, one obtains $\psi = 0.136$ and 0.134, respectively, at separation. The errors due to the approximations made in Eqs. (5) and (6) are thus fairly small. It was considered sufficiently accurate to use the step length $\psi = 0.01$ in the subsequent calculations for smaller values of K. Here separation does not occur so close to the leading edge, and the rotation has an effect on the flow, postponing separation till a higher value of ψ is reached. Thus the pressure rise between the leading edge and separation is increased. This is shown in Table 1, where ψ_s is the value of ψ at separation, θ_s being the corresponding angle in radians.

For values of K less than about 0.548, $\psi\eta_1^2$, which near the leading edge increases with increasing ψ, reaches a maximum value of less than 3 and then decreases again. Thus the separation condition is never reached, and presumably the boundary layer is stabilized completely against separation by a linear adverse external-velocity gradient.

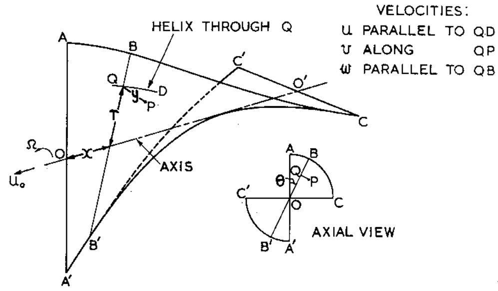

Fig. 1 Helical surface

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[3] Howarth, L., "On the solution of the laminar boundary layer equations," Proc. Roy. Soc. (London) **164A**, 547 (1938).

A Further Note on Propagation of Thermal Disturbances in Rarefied-Gas Flows

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In a recent note,[1] the small-disturbance rarefied-gas equations for one-dimensional nonsteady flow were shown to satisfy the characteristic equations

$$\left\{\frac{\partial}{\partial t} \pm 0.813\frac{\partial}{\partial x}\right\}P_{1\pm} = \left(0.487\frac{H}{p_0} \mp 0.417\frac{F}{c_0}\right)\frac{L}{c_0} + \frac{L}{t_f c_0}\left(0.15\frac{\tau}{p_0} \mp 0.4\frac{q}{p_0 c_0}\right) \tag{1}$$

$$\left\{\frac{\partial}{\partial t} \pm 2.13\frac{\partial}{\partial x}\right\}P_{2\pm} = \left(1.78\frac{H}{p_0} \pm 1.66\frac{F}{c_0}\right)\frac{L}{c_0} - \frac{L}{t_f c_0}\left(\frac{1.57\tau}{p_0} \pm 0.4\frac{q}{p_0 c_0}\right) \tag{2}$$

assuming the existence of external heat addition $H(x,t)$ and external forces $F(x,t)$ and including changes in the characteristic quantities as $t \to t_f$. The characteristic quantities $P_{1\pm}$ and $P_{2\pm}$ are defined by

$$P_{1\pm} = [(\theta - 0.51p - 0.11(\tau/p_0)] \pm (1/c_0)[0.33(q/p_0) - 0.42u] \tag{3}$$

$$P_{2\pm} = [\theta + 0.78p + 1.18(\tau/p_0)] \pm (1/c_0)[0.85(q/p_0) + 1.66u] \tag{4}$$

Received by IAS November 19, 1962; revision received December 19, 1962.

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